Journal of Engineering Physics and Thermophysics, Vol. 69, No. 4, 1996

MATHEMATICAL MODELING OF NATURAL-GAS PRODUCTION SYSTEMS

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UDC 533.551:532.546

It is proposed that a gas-bearing bed and a well be considered a single system. It is demonstrated that the equation of state of an imperfect gas can be linearly approximated.

In the description of the extraction of natural gas from trap beds through wells, the pipe gas flow and filtration of gas in a porous medium are usually considered separately, but their interrelationship is absolutely obvious. The only exception is monograph [1], where an algorithm is developed for determination of parameters in a model of gas production in terms of measured pressures and temperatures in the well mouth. In [1] a rather general mathematical model is used that takes into account the temperature nonuniformity of the gas flow in porous material and in the well and the imperfection of the gas. A similar problem was considered in [2] for an isothermal flow of a perfect gas.

At present, the use of a perfect-gas model in calculations of perfect-gas production is recognized as inadequate because of substantial increases in the depths of occurrence of gas-bearing beds, although the simplicity of obtained solutions continues to attract engineers and reserachers. Therefore, it seems useful to develop approaches that can simplify the complicated numerical algorithms for the corresponding boundary-value problems without deterioration of the accuracy of the results.

In the present article the initial problem [1] is simplified using the physics of the process and the possibility of describing the behavior of natural gas by simple functional relations, which was not noted earlier.

The first simplification of the system of equations used in [1] is that of assuming that gas filtration in the bed is isothermal. This assumption is physically reasonable, because of the high volume heat capacity of rocks in comparison with that of the filtering gas and is realized over a wide range of bed parameters, except for gas extraction from beds with very low permeability, where high pressure gradients decrease the temperature substantially in the bottom zone. For example, in monograph [1] it is shown that for a permeability of about $0.1 \cdot 10^{-12}$ m^2 neglect of the temperature nonuniformity in the gas-bearing bed results in an error of at least 1% in the determined mass flow rate. In this case, the relative decrease in the temperature is 0.05 in the bed and 0.95 in the well.

Consequently, the system of equations describing gas extraction takes the form [1]:

$$\frac{d\Pi}{dx} = x_1 \beta \frac{z(\Pi)}{\Pi}, \quad 0 < x < 1; \tag{1}$$

$$\frac{d\Pi}{dy} = -B_1 \frac{\Pi}{z(\Pi, \Theta)\Theta} - B_2 \frac{z(\Pi, \Theta)\Theta}{\Pi}; \tag{2}$$

$$\frac{d\Theta}{dy} = B_1 (\Theta_r - \Theta) - B_4 + \varepsilon(\Theta, \Pi) \frac{d\Pi}{dy},$$

$$0 < y < 1. \tag{3}$$

Institute of Physicochemical Problems of the North, Siberian Branch of the Russian Academy of Sciences Yakutsk, Russia. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 69, No. 4, pp. 540-544, July-August, 1996 Original article submitted September 14, 1993.

 1062-0125/96/6904-0420$15.00

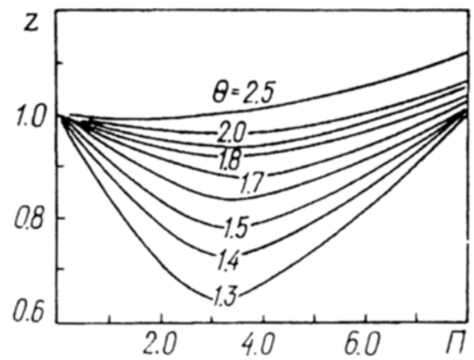

Fig. 1. Gas imperfection coefficient versus dimensionless pressure and temperature.

The boundary conditions:

$$\Pi|_{x=1} = \Pi_r, \tag{4}$$

$$\Pi|_{x=0} = \Pi|_{y=0}, \quad \Theta|_{y=0} = \Theta_r. \tag{5}$$

The dimensionless numbers and variables:

$$\Pi = p/p_{cr}, \quad \Theta = T/T_{cr}, \quad x = \ln(r/r_w)/x_1, \quad y = \bar{y}/L,$$

$$x_1 = \ln(R_b/r_w), \quad B_1 = gL/Rc_p, \quad B_2 = 8\psi M^2 RT_{cr}L/\pi^2 D^5 p_{cr}^2,$$

$$B_3 = \pi D\alpha L/c_p M, \quad B_4 = RB_1/c_p, \quad \beta = \mu MRT_r/2\pi khp_{cr}^2.$$

In problem (1)-(5) the first equation describes isothermal axisymmetric gas filtration toward the well, and Eqs. (2) and (3) describe nonisothermal gas flow in a vertical tube with allowance for heat transfer with the surrounding rocks (the first term in Eq. (3)) and gas cooling due to constriction of the flow (the first term in Eq. (3)). At the boundary of the bed the pressure is assumed to be known (Eq. (4)), and at the well bottom, the conjugation condition and the temperature uniformity condition in the bed are given (the first and second conditions of (5), respectively).

Solution of the system of Eqs. (1)-(5) is begun with integration of Eq. (1) with boundary condition (4). As a result, the quadrature is obtained

$$x_1\beta(1 - x) = \int_{\Pi}^{\Pi_r} \frac{\Pi d\Pi}{z(\Pi)}. \tag{6}$$

In principle, from (6) it is possible to determine the pressure at the well bottom assuming $x = 0$. However, it is impossible to use this result directly as a boundary condition for solution of the system of Eqs. (2) and (3).

For subsequent calculations, it is necessary to consider the behavior of the function $z(\Pi)$ at fixed $\Theta = \Theta_r$ (Fig. 1). First, it should be noted that in the practically important temperature range the function is nonmonotonic, which substantially hinders use of the most popular formulas for supercompressibility z in a wide pressure range. For example, the Berthollet equation

$$z = 1 + 0.07\Pi/\Theta_r(1 - 6/\Pi_r^2) \tag{7}$$

can be used only in the range of the dimensionless pressure in which $\partial z/\partial\Pi < 0$, i.e., on the left-hand (descending) branch of the curve $z(\Pi)$ (see Fig. 1). It is important to note that at constant temperatures formula (7) is a linear relation of the form $z = 1 + b\Pi$, where the coefficient $b < 0$, if $\Theta_r < \sqrt{6}$.

TABLE 1. Dimensionless Pressures at Well Bottom and Mouth

Case	Π_{bot}		Π_m	
	Formula (10)	Numerical solution	$L = 2000$	$L = 3000$
1	7.646	7.648	6.420	5.804
2	5.986	6.036	5.201	4.767

From the general form of the curves $z(\Pi)$ it can be assumed that they are also linear approximation for the ascending branches. It should be noted that the pressure should not exceed values at which the sign at the derivative $\partial z/\partial\Theta$ changes. It follows from Fig. 1 that for $\Theta_r = 1.8$ this limit is about $370 \cdot 10^5$ N/m^2. We use formulas of the form

$$z = z_0 (1 + b\Pi), \tag{8}$$

where the coefficients b and z_0 depend on the dimensionless temperature Θ_r of the gas-bearing bed.

Substitution of (8) into Eq. (1) with condition (4) gives

$$a(1 - x) = \Pi_r - \Pi - \frac{1}{b} \ln \frac{1 + b\Pi_r}{1 + b\Pi}, \tag{9}$$

where $a = x_1 b z_0 \beta$.

The dimensionless pressure at the bottom Π_{bot} is easily determined by formula (9). The resultant transcendental equation is solved very easily graphically. For this, it is expressed in the form

$$u - \ln u = 1 + b\Pi_r - \ln(1 + b\Pi_r) - ab, \tag{10}$$

where $u = 1 + b\Pi_w$.

At $\Pi_{bot} > 0$ the left-hand side of (10) rises monotonically from 1 to infinity. A solution of equation (10) is found at the intersection point of the curve $(u - \ln u)$ with the straight line parallel to the abscissa starting from a point on the ordinate axis equal to the right-hand side of (10).

The pressure at the bottom calculated in this way is used to determine the temperature and pressure at the well mouth; thus, initial problem (1)-(5) is reduced to integration of Eqs. (2) and (3) with boundary conditions (5), in which the mouth pressure is determined by formula (10).

The obtained results will be illustrated by calculation of gas extraction through a vertical well located in the center of a circular bed. Initial data: $R_b = 5200$, $h = 10$, $p_r = 361.5 \cdot 10^5$, $T_r = 343$ K, $L = 2000$ and 3000, $D = 2r_{cr} = 0.125$, $T_{in} = 270$ K, $\psi = 0.015$, $\alpha = 5$. The gas is methane with the following characteristics: $p_{cr} = 45.8 \cdot 10^5$, $T_b = 190.5$ K, $\mu = 0.198 \cdot 10^{-4}$, $c_p = 2093.4$, $R = 530$. Two cases are compared: 1) $k = 0.12 \cdot 10^{-12}$, $M = 9.6$; 2) $k = 0.12 \cdot 10^{-14}$, $M = 0.6785$. All the parameters are given in the SI system.

First, $\Theta = T_r/T_{cr} = 1.8$ is determined. From this isotherm in the curve $z = z(\Pi)$ (Fig. 1), the coefficients in formula (8) are determined: $z = 0.769$ and $b = 0.043$. Substitution of these values into formula (10) gives Π_{bot}, and then Eqs. (2) and (3) are integrated by the Runge–Kutta method. Results of the calculation are given in Table 1. A numerical solution is obtained by integration of Eq. (1), in which the Latonov–Gurevich formula [3] is used to determine the coefficient z. At $\Theta_r = 1.8$, it has the form

$$z = 0.83^r + 0.1p. \tag{11}$$

It can be seen from Table 1 that linear approximation (8) underestimates the results in comparison with the "exact" solution, but the error does not exceed 1%.

In conclusion, it should be noted that a similar approach can be used in integration of the equation of nonisothermal pipe flow. For example, at $\Theta = \Theta_0$ and $B_1 = 0$ (a horizontal pipeline) the formula

$$b_2 b z_0 x = \Pi_{bot} - \Pi - \frac{1}{b} \ln \frac{1 + b\Pi_{bot}}{1 + b\Pi}. \quad (12)$$

can be obtained from Eq. (2).

The relation corresponding to $B_1 \neq 0$ is not given here as it is too cumbersome.

Solution of one more practically important problem on determination of the bottom pressure in a stopped well in terms of measured pressures on the surface can also be obtained as a quadrature. For this, $B_2 = 0$ is assumed in Eq. (2) and formula (8) is used. Integration results in

$$\ln \frac{\Pi}{\Pi_0} + b(\Pi - \Pi_0) = \frac{B_1}{z_0 \Theta_r} y. \quad (13)$$

In the above formula the coordinates origin is on the surface, where the pressure $\Pi = \Pi_0$ is known. In the case of a perfect gas ($b = 0$), Eq. (13) becomes the known barometric formula.

NOTATION

p, pressure; T, temperature; r, radial coordinate with the reference point on axis of well; $\bar{y}$, coordinate along axis of well; L, well depth; R_b, coordinate of boundary of bed; g, gravity acceleration; R, gas constant; c_p, μ, specific heat at constant pressure and gas viscosity; ψ, hydraulic resistance coefficient; M, mass flow rate of gas; α, gas-rock heat transfer coefficient; k, h, permeability and thickness of gas-bearing bed. Subscripts: cr, critical parameters; r, rock; w, well; in, initial; b, boundary.

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